Introduction to Foundations and Tools Ontology Summit 2026

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Welcome

- This track will examine the foundations of ontologies from a number of perspectives.
- Invited Speakers
  - 1 April Devika P. Madalli
  - 8 April Mark Musen
  - 15 April Nicola Guarino
  - 22 April Synthesis
- I will begin with an introduction to the foundations of mathematics

Foundations of Mathematics

- The Cantor Revolution
  - Mathematics is inconsistent!
  - Foundations became a serious study
  - Set theory became the foundation of math
- Other foundation proposals emerged later
  - Ultimately still based on set theory

Category Theory

- Structure with objects and morphisms
- Abstracts common features of a wide variety of mathematical structures
- Useful for interoperability
  - Common terminology
  - Functors relate a category to another one
- Criticized as being too general
  - Derided as “abstract nonsense”
  - Eventually accepted
- Development of specializations

Internalization

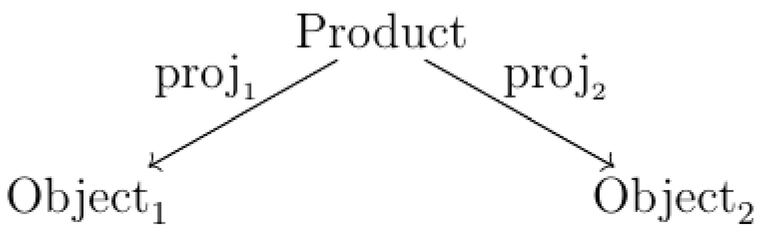
- Frequent theme in specialization of categories
- Specify concepts in terms of objects and morphisms
- Related to the process of **reification** that is a frequent theme in computer science
- Example: cartesian product
  - “External”: Set of ordered pairs
  - Internal: Object with projection morphisms

Relational Algebra

- A relation (table) is a subset of a product of types
  - The factors of the product are the columns
- Relational algebra operations
  - **Project** onto a subset of columns
  - **Join** relations to form larger relations
  - **Select** the subset of a relation that satisfies a predicate

Relational Projection Operation

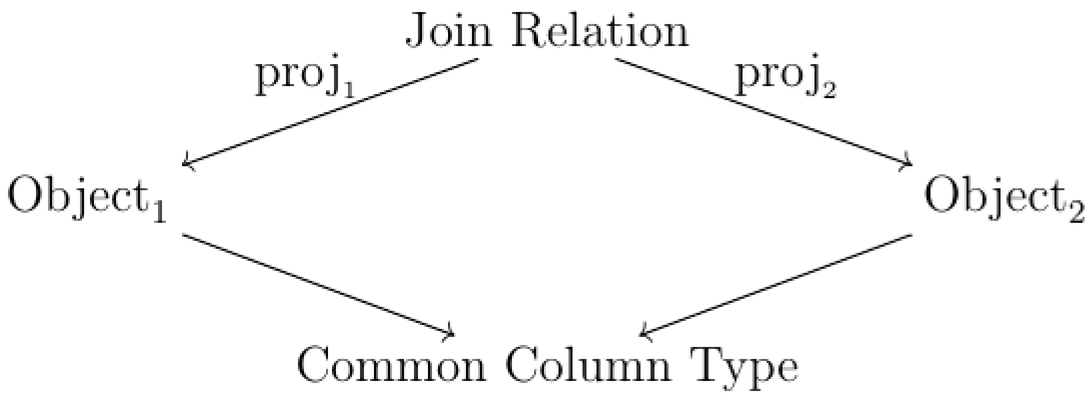
- The first requirement for a category to be a topos is that there is a product object for every pair of objects.



- The product operation is internalized in a topos using an object and two projection morphisms.

Relational Join Operation

- The simplest join is on two tables, each of which has a column with the same type



- Categories have the same notion, except that it is called a **limit**.
- The second requirement for a topos: for any finite number of objects and morphisms, there is a limit.

Relational Selection Operator

- So far a topos has project and limit, but no logic
  - This requires predicates
  - Predicates require a notion of **truth**
- Topos theory reifies truth with a *truth value object* (TVO), also called a *subobject classifier Ω*
  - Topos of sets: Ω = { true, false }
  - Topos of digraphs is in the appendix.
- The third topos requirement: it must have a TVO.

$$\text{Subobject} \xhookrightarrow[\text{inclusion}]{} \text{Object} \xrightarrow[\text{predicate}]{} \Omega$$

To Reify or not to Reify

- Advantages
  - Some second-order expressions become first-order
  - Increases applicability
  - Important for meta-programming, metamodeling, decision making, self-awareness, ...
  - Implements many forms of logic
- Disadvantages
  - Conceptually more difficult; steep learning curve
  - Different toposes and reifications may not be interoperable

Foundations

- Topos theory has the potential to be a foundation for mathematics.
- However, there are many incompatible topos theories proposed as foundation.
- For the vast majority of mathematicians, the lack of a single foundation topos is not a problem.
- Modern set theory and category theory are very deep and successful.

- A foundation ontology could serve as the foundation for ontologies.
- However, there are many (possibly) incompatible foundation ontologies.
- For many ontologists, the lack of a foundation ontology is not a problem.

Category Theory And Ancient Indian Philosophy

Modern Category Theory	Ancient Indian Philosophy	Conceptual Link
Objects defined by Morphisms	Buddhist *Pratityasamutpada*	Reality is relational; entities have no isolated internal essence.
Functors	Nyaya *Vyapti / Udaharana*	Truth-preserving mappings of structure from one domain to another.
Topos Theory	Jain *Syadvada / Anekantavada*	Truth is dependent on the specific logical universe/standpoint.
Categorical Logic	Vaisheshika *Padarthas*	The formal structuring of universals, particulars, and properties.

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Some Related Content

- K. Baclawski. Much Ado about Nothing. *J. Wash. Acad. Sci.* 109(2):13-26. (2023) pdf document
- K. Baclawski. Much Ado about Everything. *J. Wash. Acad. Sci.* 109(4):49-68. (2023) pdf document
- KGSQL Website

Appendix

- The remaining slides provide some of the details that were omitted in the presentation.

Last Topos Requirement

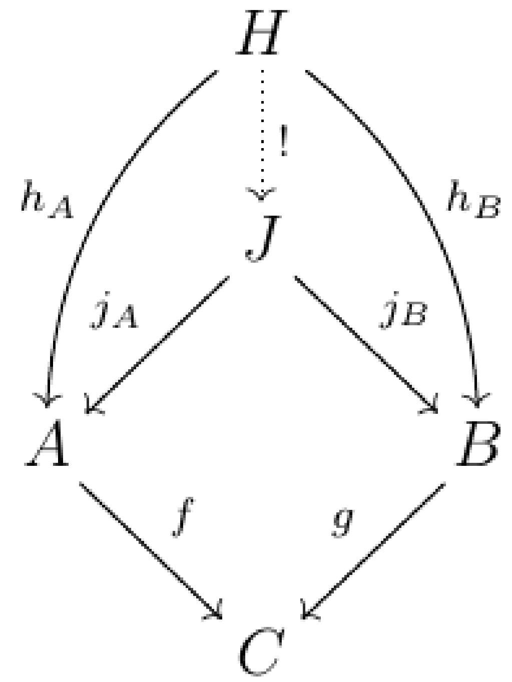
- There is one more requirement for a category to be a topos: every morphism from a product can be curried.
  - This requirement means that for objects A and B the morphisms from A to B can be reified as an object B^A

Logic

- First order logic can be interpreted in a topos
  - If the TVO is a Boolean algebra, then the logic is classical.
  - Logics that are not classical are also possible.
  - An example is in the later slides.
- Relational databases also implement first order logic.

Limit

- For objects A, B, C and morphisms f:A⟶C and g:A⟶C, a *candidate* for being the limit is an object J and morphisms (projections) j_A:J⟶A and j_B:J⟶B such that the part of the diagram below with j_A and j_B commutes. This is the join condition.
- A limit is a candidate J such that If H is another candidate then there is a unique morphism H⟶J such that the entire diagram commutes.
- The generalization to many objects and morphisms is straightforward.

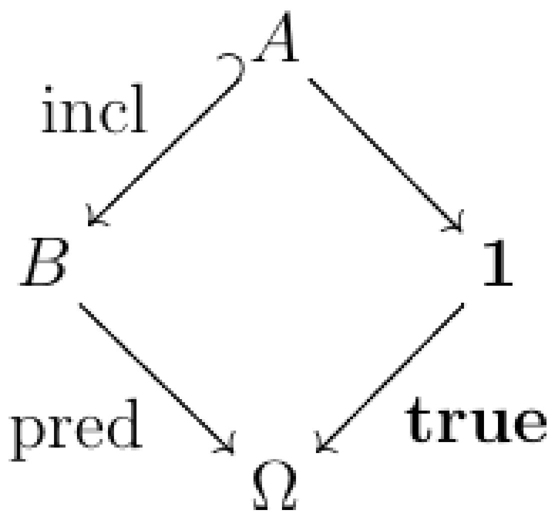


Special Limits

- The case of the limit of two objects and two morphisms is called the *pullback*. There are many other names for the pullback.
- The case where there are objects but no morphisms is a product.
- The case where there are no objects (and so no morphisms either) is usually written **1**. It has the property that for every object A there is a unique morphism A⟶**1**. Such an object is called a *final object*.
  - A morphism **1**⟶A an *element* of A.

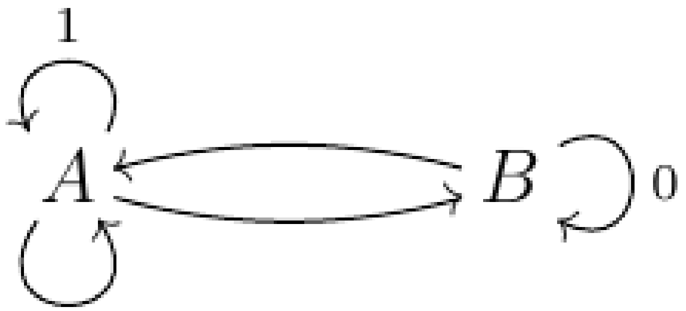
Truth Value Object

- Here is the definition of a TVO
- A TVO is actually a morphism **true**: **1**⟶Ω such that
- For every monomorphism (i.e., inclusion of a subobject in an object) incl: A⟶B there is a morphism (predicate) pred: B⟶Ω such that A is the pullback of B and Ω.
- The predicate is actually called the *characteristic morphism*.
- In other words, A consists of exactly the subobject of B for which the predicate pred has value true.



The Category of Graphs

- The category **Grph** of directed graphs is a non-Boolean topos.
- The TVO has 2 nodes and 5 edges as shown below.
- A graph homomorphism from a graph G to the TVO specifies the subgraph S such that
  - The nodes of S map to node A
  - The edges of S map to the loop labeled 1



Predicate as Explanation

- A graph homomorphism G⟶Ω not only specifies a subgraph, it also “explains” why other edges are not in the subgraph.
  - If C⟶D is in the subgraph, it maps to the loop labeled 1
  - If C and D are in the subgraph but C⟶D is not, then C⟶D maps to the other loop at A
  - If C is in the subgraph but D is not, then C⟶D maps to A⟶B
  - If D is in the subgraph but C is not, then C⟶D maps to B⟶A
  - If Neither C nor D is in the subgraph, then C⟶D maps to the loop labeled 0